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▶ To cite this version:

Mohammed Ziani, Régis Duvigneau, Michael Doerfel. On the role played by NURBS weights in isogeometric structural shape optimization. International Conference on Inverse Problems, Control and Shape Optimization, Apr 2010, Cartagena, Spain. inria-00535789

HAL Id: inria-00535789

https://hal.inria.fr/inria-00535789

Submitted on 12 Nov 2010

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V International Conference on Inverse
Problems, Control and Shape Optimization
Cartagena (Spain)
April 7-9, 2010

On the role played by NURBS weights in isogeometric structural shape optimization

Mohammed Ziani[1], Régis Duvigneau[1] and Michael R. Dörfel[2]

[1] INRIA Sophia-Antipolis Méditerranée Opale Project-Team
2004 route de lucioles 06902 Sophia-Antipolis, FRANCE
e-mail:{Mohammed.Ziani,Regis.Duvigneau}@sophia.inria.fr

[2] Technische Universität München
D-85748 Garching bei München, Deutschland
e-mail:doerfel@ma.tum.de

Abstract

Finite element based shape optimization is a classical approach for solving realistic problems, such as those encountered in industry. However, some difficulties may arise from the divide between the parametrization of the shape using Non-Uniform Rational B-Spline (NURBS) functions used in CAD tools and the finite element representation of the geometry using piecewise linear functions used in solvers. An alternate approach based on *isogeometric analysis* has been recently proposed for which the solver is also based on NURBS functions. In this framework, the locations of NURBS control points are usually considered as optimization variables whereas the corresponding control weights are frozen. However, this *a priori* choice of weights imposes a severe limitation of the shapes that could be found by the optimization algorithm. In this work, we present and experiment a shape optimization algorithm where the weights are taken as optimization variables in addition to control points.

Key words: Structural shape optimization, isogeometric analysis, NURBS weights.

1 Introduction

Structural shape optimization consists in searching for the geometry that minimizes a cost function, like mass or compliance, subject to some

mechanical loading [4, 5]. In a structural shape optimization process, the finite element method is usually employed within the analysis model to compute the structural response, whereas the geometry parametrization is described using NURBS functions. However, many difficulties can arise when separating the design model and the analysis model. Wall et *al.* propose in [10] an alternate approach based on *isogeometric analysis*.
In isogeometric analysis, developed at first by Hughes et *al.* [6], both the solution space and the computational domain are represented as NURBS functions, yielding the integration of finite element and Computer Aided Design (CAD) methods. Isogeometric analysis has several advantages over the standard finite element analysis such as *geometric exactness* and *simple refinement*. For more details about NURBS, see [8, 9].
The choice of design variables can be critical for the success of the optimization process. In [10], authors consider the locations of NURBS control points as optimization variables whereas the corresponding control weights are frozen. However, this *a priori* choice of weights imposes a severe limitation of the shapes that could be found by the optimization algorithm. We propose in this work to use NURBS weights as optimization variables in addition of control points. Tests will be done on a two-dimensional linear elasticity problem.

2 Isogeometric analysis

Isogeometric analysis is a generalization of classical finite element analysis. It consists in an isoparametric analysis approach where basis functions generated from NURBS are employed in order to describe both geometry and unknown variables of the problem [6]. We recall here some definitions of isogeometric analysis background.

2.1 NURBS basis functions

NURBS are based on B-spline basis functions. Consider a *knot vector* Ξ in one dimensional space, which is a set of coordinate ξ_i in a parametric space:

$$\Xi = \{\xi_1, \xi_2, ..., \xi_{n+p+1}\},$$

with $\xi_1 \leq \xi_2 \leq ... \leq \xi_{n+p+1}$. Here p and n are the degree and the number of basis functions, respectively. The n univariate B-spline basis functions of degree p are defined recursively, see [2], by:

$$N_{i,0} = \begin{cases} 1 & \text{if } \xi_i \leq \xi < \xi_{i+1} \\ 0 & \text{otherwise} \end{cases}$$

$$N_{i,p}(\xi) = \frac{\xi - \xi_i}{\xi_{i+p} - \xi_i} N_{i,p-1}(\xi) + \frac{\xi_{i+p+1} - \xi}{\xi_{i+p+1} - \xi_{i+1}} N_{i+1,p-1}(\xi), \quad i = 1, ..., n+p+1. \tag{1}$$

We use non-uniform open knot vectors where the first and the last knot are repeated $(p+1)$ times. B-spline curves of degree p are obtained from the linear combination of B-spline basis-functions of degree p and the corresponding control points P_i, $i = 1, ..., n$:

$$C(\xi) = \sum_{i=1}^{n} N_{i,p}(\xi) P_i. \tag{2}$$

NURBS are rational B-spline curves which are the projection of a B-spline curve $C^w(\xi)$ defined in $(d+1)$-dimensional homogeneous coordinate space back onto the d-dimensional physical space $\mathbb{R}^d$. Homogeneous weighted $(d+1)$-dimensional control points are $P_i^w = (w_i x_i, w_i y_i, w_i)^T$. The $(d+1)$-dimensional B-spline curve C^w then reads

$$C^w(\xi) = \sum_{i=1}^{n} N_{i,p}(\xi) P_i^w.$$

Projecting onto $\mathbb{R}^d$ by dividing through the additional coordinate yield the *rational* B-spline curve:

$$C(\xi) = \frac{\sum_{i=1}^{n} N_{i,p}(\xi) w_i P_i}{\sum_{i=0}^{n} N_{i,p}(u) w_i} = \sum_{i=1}^{n} R_{i,p}(\xi) P_i, \tag{3}$$

where $P_i, \; i = 1, ..., n$ are the control points, $w_i \geq 0, \; i = 1, ..., n$ are the weights, and $R_{i,p}(\xi)$ are the *rational basis functions*. The weights modify the influence of the control points on the curve.

A NURBS surface is obtained by taking a bidirectional net of control points, two knot vectors, and the tensor product of two univariate NURBS functions

$$S(\xi, \eta) = \sum_{i=1}^{n} \sum_{j=1}^{m} \frac{N_{i,p}(\xi) N_{j,q}(\eta) w_{i,j}}{\sum_{i=1}^{n} \sum_{j=1}^{m} N_{i,p}(\xi) N_{j,q}(\eta) w_{i,j} w_{i,j}} P_{i,j}.$$

2.2 Isogeometric analysis of elastic problems

In this section we are interested in discretizing and computing the deformation of a two dimensional solid body. The solid lies in the domain $\Omega \subset \mathbb{R}^2$ and is characterized by the displacement field $u(x) \in \mathbb{R}^2, \; x \in \Omega$. The boundaries are composed of a prescribed displacement boundary Γ_u and a prescribed traction boundary Γ_t with $\Gamma = \partial\Omega = \Gamma_u \cup \Gamma_t$ and $\Gamma_u \cap \Gamma_t = \emptyset$. The body is subject to the body force intensity b and the prescribed traction $\bar{t}$ on Γ_t.

Under the assumption of small deformations, the solid response is described in terms of the linear strain tensor $\epsilon(u) = \left(\nabla u + \nabla u^T\right)/2$. The governing equation for the displacement u with imposed Dirichlet boundary conditions $u = \bar{u}$ on Γ_u reads in weak form

$$\int_\Omega \delta\epsilon : \sigma \; d\Omega = \int_\Omega \delta u.b \; d\Omega + \int_{\Gamma_t} \delta u.\bar{t} \; d\Gamma. \tag{4}$$

Here, we denote by σ the stress tensor given by $\sigma_{ij}(u) = 2\mu\epsilon_{ij}(u) + \lambda \, \mathrm{tr}(\epsilon(u))\mathbb{I}_2, \quad i,j = 1,2$, where λ and μ denote Lamé constants.

Isogeometric analysis adopts the same procedure as finite element analysis to find a solution u, but the discretization (both geometry and solution) is now based on NURBS instead of polynomials as shape functions. Hence, the discretized geometry and solution are given by:

$$x(\xi,\eta) = \sum_{i=1}^{n}\sum_{j=1}^{m} N_{i,p}(\xi)N_{j,q}(\eta)P_{i,j} \quad \text{and} \quad u(\xi,\eta) = \sum_{i=1}^{n}\sum_{j=1}^{m} N_{i,p}(\xi)N_{j,q}(\eta)d_{i,j}.$$

3 Isogeometric structural shape optimization

In isogeometric structural shape optimization, the boundary control points usually play the role of design variables. We seek to solve the structural shape optimization

$$\begin{array}{cc} \min_y z(y, u(y)), & y \in \mathbb{R}^n \\ \text{subject to} & V = V_0, \end{array} \tag{5}$$

where V and V_0 are the volume and the initial volume of the structure, respectively and x are the design variables with n components. The state variable u describes the structural response, e.g. the displacements. z is the objective function (in our case the compliance) and its discrete form is written $z = f^T u$, with u as displacement vector and f as external force vector. The constraint appearing in the definition of the optimization problem (5) is taken into account by means of a suitable *penalty term* $\varepsilon > 0$. Then we minimize the unconstrained functional

$$z + \varepsilon \left(V - V_0\right)^2 .$$

The problem (5) is solved in our tests by the conjugate gradient method, see [7], where the gradient is computed by finite differences approximation. Contrary to [10], control points as well as weights are considered as design variables.

4 Numerical results

In our tests, we focus on the shape optimization of a hole in a large plate under a biaxial stress. Due to symmetry, only a quarter of the plate is modeled. We seek for an optimal shape of minimal compliance subject to a volume constraint. The initial isogeometric discretization, consisting of one

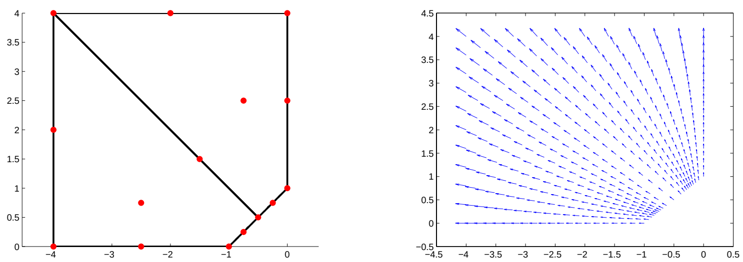

Figure 1: NURBS discretization of initial geometry (left) and initial displacement field (right).

NURBS patch of two elements, and the initial displacement field are shown in figure 1. The NURBS surface of biquadratic order is defined by the open knot vectors $\Xi_1 = \{0,\ 0,\ 0,\ 0.5,\ 0.5,\ 1,\ 1,\ 1\}$ and $\Xi_2 = \{0,\ 0,\ 0,\ 1,\ 1,\ 1\}$ which yield 15 control points. Taking only the control points as optimization variables can be critical to the optimization process. In fact, weights have a crucial geometric meaning to define NURBS. At first time, we suppose that weights are not optimization variables and are fixed *a priori*. Figure 2 presents the shape obtained for two different choices of weights.

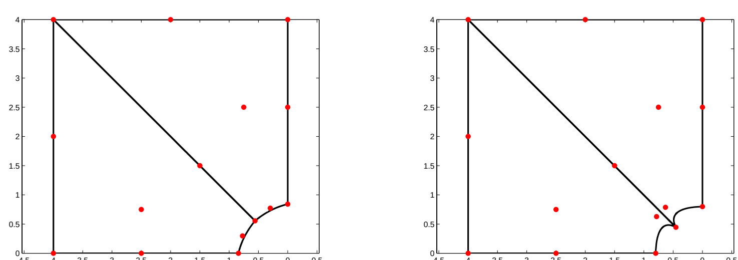

Figure 2: Result design of two set values of weights, left: $w_i = 1,\ i = 1, .., 5$, right: $w_1 = 1, w_2 = 2, w_3 = 5, w_4 = 2, w_5 = 1$.

Now, we consider weights as optimization variables. The optimal design and the final displacement field are depicted in figure 3. The convergence of the optimization algorithm is improved in this case as well as the final cost function value. We present the reduction of the objective function in figure 4.

One should underline that, in the isogeometric analysis context, the domain parameterization and the solution fields are not independent. In particular, any change of the parameterization modifies the physical solution obtained. A possible drawback of the proposed approach is the increase of the optimization problem dimension. Therefore, we are now studying how to adjust the weights during the optimization procedure, without considering them as design variables.

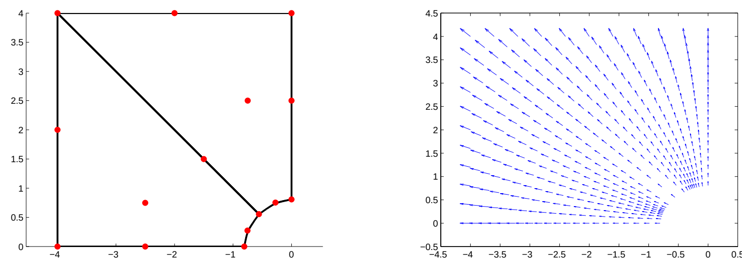

Figure 3: Optimal design (left) and final displacement field (right).

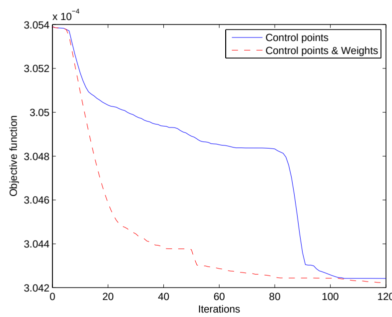

Figure 4: Optimization history.

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